Soler Garrido, J., & Piechocki, RJ. (2007). Analog implementation of a mean field detector for multiple antenna systems. *IEEE International Symposium on Circuits and Systems (ISCAS 2007)*, 3880 - 3883. https://doi.org/10.1109/ISCAS.2007.377886

Analog Implementation of a Mean Field Detector for Multiple Antenna Systems

Josep Soler Garrido, Robert J. Piechocki
Centre for Communications Research
University of Bristol
Bristol, UK
Josep.Soler@bristol.ac.uk

Abstract— **In this paper we present an analog implementation of the mean-field algorithm for inference in Undirected Graphical Models (UGM). One of its numerous applications is signal detection in wireless receivers which can benefit from low power analog circuitry. An example detector for a double Alamouti Space-Time Block Code (STBC) has been designed and laid out in a 0.25µm SiGe BiCMOS process.**

I. Introduction

In this contribution we consider an inferential task on Undirected Graphical Models, also known as Markov Random Fields (MRF). UGM are a very useful formalism for describing conditional dependence between random variables. An UGM consists of a graph G with an associated set of variables and connecting edges $G=\{V, E\}$. An example of such graph is depicted in Fig.1, where the larger red dots represent unobserved variables of interest and smaller blue dots represent observed variables. The structure of the graph (connections) lends itself to a factorization of the joint distribution function

$$f(\mathbf{x}) \propto \prod_{s\in V} \varphi_s(x_s) \prod_{(s,t)\in E} \varphi_{s,t}(x_s, x_t) \ . \qquad (1)$$

From equation (1) we note that the joint distribution of interest is a product over cliques (sets of fully connected nodes), which are singleton variables and pairs of variables. Our task is to calculate sets of marginal distributions for all unobserved variables, given the values of the observed variables and the structure of the graph, i.e.

$$f(x_i) = \frac{1}{Z} \sum_{s_{-i}\in V} \prod_{s\in V} \varphi_s(x_s) \prod_{(s,t)\in E} \varphi_{s,t}(x_s, x_t) . \qquad (2)$$

where Z is a normalizing constant and "$-i$" stands for "all except i".

This task can be efficiently performed (either exactly or resorting to approximations) with analog circuitry. Analog signal processing has the potential to outperform digital implementations in terms of power consumption and silicon area, as a great number of nonlinear operations and algorithms can be mapped in analog circuits in a straight-forward manner. Analog decoding of channel codes [1] is a good example of inference on graphs in analog domain with applications in the field of communications. These decoders typically perform the computation of approximate marginal distributions using sum-product [2] or belief propagation algorithm which can be very efficiently implemented with translinear circuits. Graphical models for error correcting codes tend to contain a low density of connections. The advantages of analog implementation of decoders for such graphs are twofold. First, message passing algorithms, which can be highly parallelized, offer near optimal performance on them. Secondly, the low degree of interconnections ensures that resulting layouts are very compact. If we move beyond sparse graphs, sum-product and related algorithms start to fail due to the presence of short cycles or loops in the graphs, especially if the interactions between variables are strong.

In this paper we try to extend the application of such analog decoders to include inference on graphs with a higher degree of interconnections. Analog implementation of naive Mean Field [3, 4] algorithm is proposed. Due to the simplicity of the approximation used, complexity (and thus silicon area) can be reduced compared to existing sum-product decoders. Simulated annealing is proposed alleviate a possible performance degradation caused by short loops and strong variable interactions.

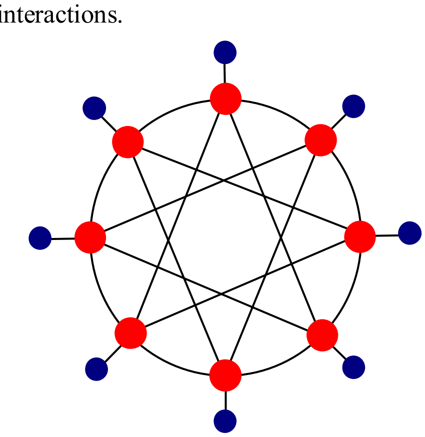


Figure 1. UGM of the considered problem.

Research sponsored by Toshiba TRL

1-4244-0921-7/07 $25.00 © 2007 IEEE.

The rest of the paper is organized as follows. Section II introduces the Mean Field algorithm and its application to MIMO detection. Section III outlines analog implementation. In section IV the design of an example detector is presented. Finally, conclusions are drawn in section V.

II. MEAN FIELD INFERENCE

Our generic inference problem can be modeled as

$$\mathbf{z} = \mathbf{R}\mathbf{x} + \mathbf{v} \tag{3}$$

where $\mathbf{x} = (x_1, \ldots x_N)^T$ is the set of unobserved variables for which marginal distributions have to be calculated, $\mathbf{z}$ is a vector containing the observations, and the matrix $\mathbf{R}$ describes the interactions between variables (i.e. defines the strengths of the edge potentials of the UGM of interest), with $\mathbf{v}$ being a term of non-white Gaussian noise $\mathbf{v} \sim \mathcal{N}(0, \sigma_n^2\mathbf{R})$. In the sequel we will assume the terms of $\mathbf{x}$ take values from a binary alphabet $\{-a, a\}$. From (3) we can write

$$z_i = r_{i,i} + \sum_{j=1, j\neq i}^{n_i} r_{i,j} x_j + v_i \tag{4}$$

where z_i, r_{ij} and v_i are the terms of $\mathbf{z}$, $\mathbf{R}$ and $\mathbf{v}$ respectively, and n_i is the number of connections to variable i in the graphical model. It can be noted that each observation is a sum of a term containing the variable of interest plus an undesired term which contains interference from the rest of variables and noise. The variable's posterior mean value can be obtained from equation (5)

$$E\{x_i | z_i, x_{-i}\} = a \frac{\exp(\mu a/\sigma_r^2) - \exp(-\mu a/\sigma_r^2)}{\exp(\mu a/\sigma_r^2) + \exp(-\mu a/\sigma_r^2)} \tag{5}$$

where

$$\mu = \frac{1}{r_{i,i}}\left(z_i - \sum_{j=1, j\neq i}^{n_i} r_{i,j} x_j\right), \; \sigma_r^2 = \frac{1}{r_{i,i}}\sigma_n^2 .$$

In mean field algorithm, the values for the posterior means are successively approximated as

$$\hat{m}_i = E\{x_i | z_i, \hat{m}_{-i}\} = a \cdot \tanh\left(\frac{a}{\sigma_n^2}\left(z_i - \sum_{j=1, j\neq i}^{n_i} r_{i,j}\hat{m}_j\right)\right). \tag{6}$$

In order to improve the convergence we can resort to simulated annealing [5], leading to our final expression:

$$\frac{\hat{m}_i}{a} = \tanh\left(\frac{T}{T_{\min}}\left(\bar{z}_i - \sum_{j=1, j\neq i}^{n_i} \bar{r}_{i,j}\hat{m}_j\right)\right) \tag{7}$$

where T is a normalized annealing parameter which increases gradually from zero to unity and $T_{\min}$ represents the temperature in the last iteration (i.e. the lowest temperature, which should be of the same order of magnitude as σ_n^2). For convenience, the elements of $\mathbf{z}$ and $\mathbf{R}$ have also been normalized, i.e. $(|\bar{z}_i|, |a \cdot \bar{r}_{i,j}| < 1)$.

In section III, analog implementation of equation (7) is discussed.

As aforementioned, the application targeted is detection of Space Time Block Codes (STBC). Specifically, a simple Alamouti STBC is considered [6], in which diversity is obtained by transmitting two symbols s_0, s_1 in two time slots and using two antennas. Assuming QPSK modulation, the transmission scheme can be summarized as:

$$s_0 = x_1 + j \cdot x_2, \qquad s_1 = x_3 + j \cdot x_4$$

$$\mathbf{s}_t = \begin{bmatrix} s_0 \\ s_1 \end{bmatrix}, \qquad \mathbf{s}_{t+T} = \begin{bmatrix} -s_1^* \\ s_0^* \end{bmatrix}$$

In this case, if the channel remains static during both time slots, matched filter becomes optimal in the ML sense and detection complexity is linear ($\mathbf{R}$ is diagonal). We will consider a combination of two Alamouti transmitters working in parallel [7], therefore doubling the rate at the expense of sacrificing orthogonality. The resulting 8×8 matrix $\mathbf{R}$ has the form:

$$\mathbf{R} = \begin{bmatrix} k_1\mathbf{I}_4 & \mathbf{A} \\ \mathbf{A}^T & k_2\mathbf{I}_4 \end{bmatrix}$$

where $\mathbf{I}_4$ is the identity matrix of size four, and $\mathbf{A}$ is a 4×4 matrix of non-zero elements which represent interactions between non-orthogonal variables. Therefore, only two quadrants are diagonal whereas the other two are fully connected. This particular inference problem can also be represented by the UGM of Fig.1, where each variable is connected to four others. This kind of graph already becomes difficult for message passing algorithms due to the short loops present, especially when the number of receive antennas approaches the minimum (two in this case).

Bit error rate curves for the proposed algorithm applied to this system are presented in Fig. 2, for the cases of two, three and four receive antennas. As diversity is increased by adding more receive antennas, the interactions between variables become weaker (off-diagonal elements of $\mathbf{R}$ have lower values) and mean field algorithm becomes near optimal.

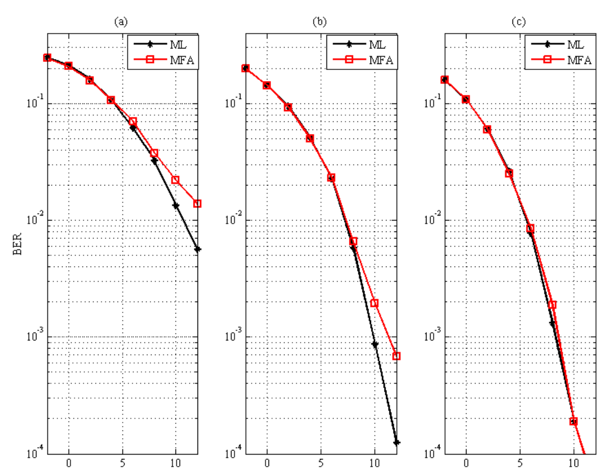


Figure 2. Optimal and mean field BER curves for the proposed system for a) 2 receive antennas, b) 3 receive antennas, c) 4 receive antennas.

III. ANALOG IMPLEMENTATION

As previously noted, the term inside the *tanh* function in (7) contains a local observation and a sum of terms which account for the interference from other variables. Computation of these terms is straightforward in analog domain by using a modified version of a well known circuit: the Gilbert multiplier [8]. A single Gilbert multiplier like the one in Fig. 3 can efficiently compute the product of two mean values.

In our case, the two sets of input currents are made proportional to the corresponding element $r_{i,j}$ of the inference matrix and the current estimate $\hat{m}_j$ respectively:

$$I_{X1} - I_{X0} = I_{ref} \cdot a \cdot \overline{r}_{i,j}, \qquad I_{Y1} - I_{Y0} = I_{ref} \cdot \hat{m}_j / a$$

that is:

$$I_{X0} = \frac{I_{ref}}{2}\left(1 - a \cdot \overline{r}_{i,j}\right), \qquad I_{X1} = \frac{I_{ref}}{2}\left(1 + a \cdot \overline{r}_{i,j}\right),$$

$$I_{Y0} = I_{ref}\, p\left(x_j = -a\right), \qquad I_{Y1} = I_{ref}\, p\left(x_j = a\right).$$

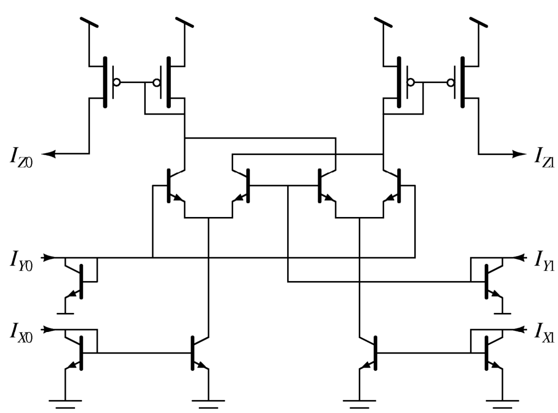
Note that since we normalized the terms of the matrix, all the input currents take some positive value smaller than the reference current I_{ref}. The value of I_{ref} will be chosen taking into account the required decoding speed and maximum power consumption of the circuit.

This leads to the following output currents for the Gilbert multiplier:

$$I_{Z0} = \frac{I_{ref}}{2}\left(1 - \overline{r}_{i,j} \cdot \hat{m}_j\right), \qquad I_{Z1} = \frac{I_{ref}}{2}\left(1 + \overline{r}_{i,j} \cdot \hat{m}_j\right)$$

and thus the output differential current is proportional to the desired interference term. It is worth noting that this current domain representation of the terms is particularly convenient since current signals can be summed without any additional circuitry.

$$I_{Z1} - I_{Z0} = I_{ref} \cdot \overline{r}_{i,j} \cdot \hat{m}_j . \tag{8}$$



$$I_{Z1} = (I_{X0} I_{Y0} + I_{X1} I_{Y1}) \;/\; (I_{Y0} + I_{Y1})$$
$$I_{Z0} = (I_{X0} I_{Y1} + I_{X1} I_{Y0}) \;/\; (I_{Y0} + I_{Y1})$$

Figure 3. Schematic of a Gilbert current multiplier and output current equations.

Therefore, by simply connecting together the outputs of several multipliers we can obtain a set of currents $\{I_1, I_0\}$ such that

$$I_1 - I_0 = I_{ref}\left(\overline{z}_i - \sum_{j=1, j\neq i}^{n_i} \overline{r}_{i,j} \cdot \hat{m}_j\right) \tag{9}$$

These currents are then passed to the circuit of Fig. 4. The first stage is an additional Gilbert multiplier which performs simulated annealing. This makes possible a single stage implementation of the algorithm since annealing can be performed 'on the fly', i.e. the temperature can be controlled directly from an external signal:

$$I_{A0} = \frac{I_{ANN}}{2}\left(1 - T\right), \qquad I_{A1} = \frac{I_{ANN}}{2}\left(1 + T\right).$$

The difference of voltage between the terminals of the resistors becomes

$$\Delta V = I_{ANN} \cdot R \cdot T \cdot \frac{I_1 - I_0}{\left(n_i + 1\right) \cdot I_{ref}} \tag{10}$$

Finally, the output differential pair performs the *tanh* function required to compute a new update of the posterior mean:

$$I_{m1} - I_{m0} = I_{ref} \tanh\left(\frac{I_{ANN} \cdot R \cdot T}{2U_T\left(n_i + 1\right)}\left(\overline{z}_i - \sum_{j=1, j\neq i}^{n_i} \overline{r}_{i,j} \cdot \hat{m}_j\right)\right). \tag{11}$$

In a complete decoder, the number of Gilbert multipliers is determined by the number of connections to the each variable in the graphical model. In our case, n_i=4 for all variables, so four multipliers per variable are needed, and eight individual blocks make the complete Mean Field detector. In general, if the density of interconnections remains constant then the complexity of the algorithm (and the size of the circuit) is $O(N^2)$ where N is the number of variables.

Other message passing algorithms, like sum-product, have the same order of complexity but besides Gilbert multipliers they require additional circuitry to perform summation of log-probabilities in voltage domain, whereas in our case the summations are performed in current domain. On the downside, mean field requires annealing to offer the same level of performance, therefore making convergence slower. In the analog implementation that means longer transients can be expected before the currents settle to a good estimate.

IV. ANALOG MIMO DETECTOR CHIP

Layout of the complete detector is shown in Fig. 5. The analog part consists (from bottom to top of the layout) of a matrix of 32 Gilbert multipliers (one for each non-zero off-diagonal element of **R** or, equivalently, two for each edge on the UGM), a current normalization stage, 8 additional multipliers to perform simulated annealing, and a final stage performing *tanh* function.

While all the inputs to the circuit are purely analog (with the exception of reset signals to bring all the initial probabilities to a uniform distribution), comparators have been added to provide hard (digital) decisions at the output.

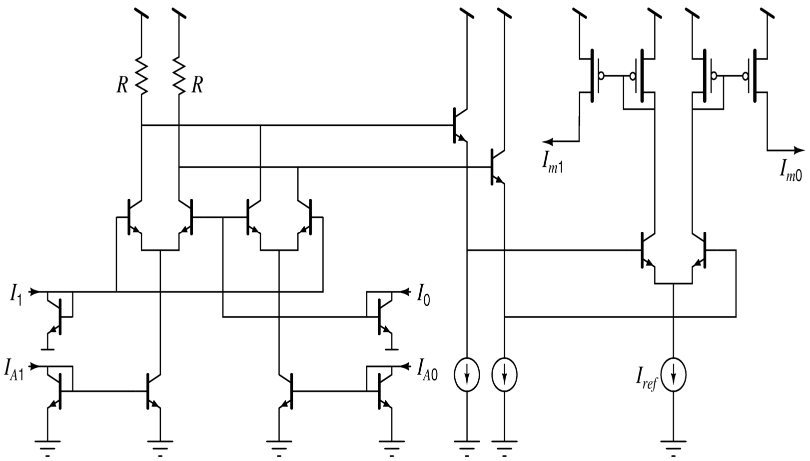


Figure 4. Output block performing simulated annealing and non-linear function.

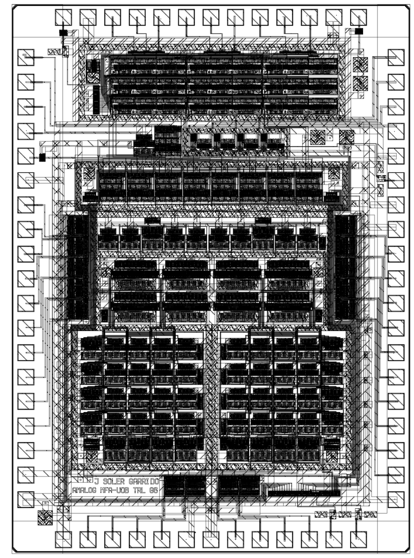


Figure 5. Layout of the complete detector

The circuit has been designed in a 0.25µm SiGe BiCMOS process and operates from a single supply of 3.3V. The estimated power consumption of the analog core is 140mW, and simulations show a potential decoding speed of over 100 Mbps. The total area of the chip is 5.2 mm^2. It is worth noting that due to the fully parallel implementation decoding speed can increase linearly with the number of variables, thus making possible a very high throughput for more complex systems.

Although HBTs were used for this particular design, translinear circuits can also be constructed employing MOS transistors operated in weak inversion, which exhibit a similar exponential voltage-current characteristic. A purely MOS implementation would effectively reduce the power consumption and total area of the decoder.

The choice of bipolar transistors is based on matching considerations. Simulations including models for transistor mismatch show that the degradation in performance is a mere shift to the right on the bit error rate curves, unless matching becomes poor enough to create an error floor above the inherent mean field one. Typical mismatch in sub-threshold MOS transistors dangerously approaches this limit for our application to MIMO cases where performance relies on accurate representation of variable and edge potentials. For other applications (e.g. channel decoders, or a combination of MIMO detection and channel decoding), a purely MOS implementation would be viable since the whole system is somewhat more immune to transistor mismatch.

V. Conclusions

Inference in analog circuits based on mean field annealing has been proposed. The main benefit of the mean field approximation is its simplicity which translates directly on simpler layouts compared to existing analog decoders based on sum-product algorithm, thus making it more suitable for factor graphs with a high degree of connections.

A decoder prototype has been designed and laid out with the objective of testing the proposed implementation. The circuit is able to decode signaling from two parallel Alamouti transmitters. The performance of the analog detector and effects of process variations and transistor mismatch will be evaluated on real silicon.

Acknowledgment

The authors would like to thank Dr. Darren McNamara for his help in all aspects of this project, and the directors of Toshiba TRL for an excellent support. The first author would also like to thank IHP microelectronics for their help in designing the chip.

References

[1] H.A. Loeliger, F. Lustenberger, M. Helfenstein, and F. Tarkoy, “Probability propagation and decoding in analog VLSI.” *IEEE Trans. Inform. Theory,* vol. 47, pp. 837–843, Feb. 2001.

[2] K. Murphy, Y. Weiss, and M. Jordan, “Loopy-belief propagation for approximate inference: An empirical study.” *Proc. of the 16th Conference on Uncertainty in Artificial Intelligence*, pp. 467--475.

[3] M. J. Wainwright and M. I. Jordan “A variational principle for graphical models.” *New Directions in Statistical Signal Processing: From Systems to Brain*. Cambridge, MA: MIT Press, 2005.

[4] M. Opper and D. Saad “Advanced Mean Field Methods, theory and practice” The MIT press, 2001.

[5] S. Kirkpatrick, C.D. Gelatt, and M.P. Vecchi. “Optimization by simulated annealing.” *Science*, (2): pp. 671--680, 1983.

[6] S.M Alamouti, “A simple transmit diversity technique for wireless communications.” *IEEE J. Select. Commun*., vol. 16, No. 8, pp. 1451–1458, Oct. 1998.

[7] Naguib, A.F.; Seshadri, N.; Calderbank, A.R.; "Increasing data rate over wireless channels" *IEEE Signal Processing Magazine*, Volume 17, Issue 3, pp. 76–92, May 2000.

[8] B. Gilbert. “A precise four-quadrant multiplier with subnanosecond response.” *IEEE Journal of Solid-State Circuits*, vol. 3, pp. 365–373, 1968.